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On the Stability of the Classical Vacua in a Minimal SU(5) 5-D Supergravity Model.

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Abstract

We consider a five-dimensional supergravity model with $SU(5)$ gauge symmetry and the minimal field content. Studying the arising scalar potential we find that the gauging of the $U(1)_R$ symmetry of the five-dimensional supergravity causes instabilities. Lifting the instabilities the vacua are of Anti-de-Sitter type and $SU(5)$ is broken along with supersymmetry. Keeping the $U(1)_R$ ungauged the potential has flat directions along which supersymmetry is unbroken.

1 Introduction

It is well established that the Standard Model (SM) describes succesfully all particle interactions at low energies. On the other hand it is understood that SM is an effective theory. At higher energies, the description of the elementary particle interactions demands a generalization of the SM. Assuming a unified description in terms of a renormalizable field theory, up to very high energies, lead to favorable generalization namely GUT theories [1], among which supersymmetric GUTs [2] play a central rôle. Consistent inclusion of gravity dictates that these generalizations should be effective descriptions of a more fundamental underlying theory and String Theory [3, 4] is the most prominent candidate for this aim. Indeed from the 10 dimensional field theory, which is the effective point limit of the String Theory we can get, by suitable compactifications of the extra dimensions, consistent four-dimensional models compatible with the SM [5]. Along these lines it has been conjectured that one or two dimensions may be compactified at different scales, lower from the remaining ones [6]. Also, after the developments concerning the duality symmetries of String Theory and in the framework of M-Theory [7, 8], the idea that our world may be a brane embedded in a higher dimensional space has recently attracted much interest and has been studied intensively [9, 10, 11, 12, 13, 14, 15]. Besides the original compactifications new possibilities have been proposed [16]. It has been also recognized that String/M theory may lead to brane-world models in which one of the extra dimensions can be even non-compact [17, 18]. In all these models the four-dimensional world is a brane, on which the matter fields live, while gravity, and in some interesting cases the gauge and the Higgs fields, propagate also in the transverse extra dimensions of the bulk space.

In the majority of the cases studied, in an attempt to build realistic models, the bulk is a five-dimensional space [19]. In these models the corresponding backgrounds may be of Minkowski or Anti-de-Sitter type. Effects of the above considerations in specific GUT models have been also considered. The assumed background in these models is of Minkowski type and questions regarding the unification and supersymmetry breaking scales have been addressed to. In this direction, assuming the fifth dimension very large, of the TeV scale, non supersymmetric extensions of the SM even without the need of unification have been considered [20, 21, 22, 23, 24, 25]. On the other hand, models embedding the SM in an Anti-de-Sitter five dimensional space have been also discussed [26, 27].

In view of the aforementioned developments the study of the five dimensional supergravities has been revived [28, 29, 30, 31, 32, 33]. This is quite natural since after all gravity is in the center of all these attempts and it is legitimate to assume that we have to treat the fifth dimension before going to the "flat limit". In the framework of the five-dimensional supergravity no specific model based on a particular gauge group has been studied so far.

In this work we consider five dimensional supergravity with the minimal unified gauge group, $SU(5)$. The gauge symmetry enters with a minimal number of multiplets following the construction of [29]. We study the emerging potential and discuss the Minkowski or Anti-de-Sitter character of the classical vacua. We comment on the fate of the gauge symmetry and supersymmetry of these vacua, ignoring at this stage the inclusion of matter fields, which do not play any rôle in this consideration.

2 Setting the model

We consider a five-dimensional gauged supergravity model with $SU(5)$ gauge symmetry. The field content of the model is [28]

$$\{e_\mu^m, \Psi_\mu^i, A_\mu^I, B_{\mu\nu}^M, \lambda^{i\tilde{a}}, \varphi^{\tilde{x}}\} \tag{1}$$

where

$$\begin{aligned} I &= 0, 1, \ldots, n \\ M &= 1, \ldots, 2m \\ \tilde{a} &= 1, \ldots, \tilde{n} \\ \tilde{x} &= 1, \ldots, \tilde{n} \end{aligned}$$

with $\tilde{n} = n + 2m$. The supergravity multiplet consists of the fünfbein e_μ^m, two gravitini Ψ_μ^i and the graviphoton A_μ^0, where $i = 1, 2$ is the symplectic $SU(2)_R$ index. Moreover, there exist 25 vector multiplets, $n = (a, s)$, $a = 1, \ldots, 24$, including the $SU(5)$ gauge fields (A^a_μ) plus one $SU(5)$ singlet, labelled by "s", which is introduced for reasons that will be explained later. The field content is completed by ten tensor multiplets, $M = 1, \ldots, 10$, transforming as $5 + \bar{5}$ under $SU(5)$. The spinor and the scalar fields included in the vector and tensor multiplets are collectively denoted by $\lambda^{i\tilde{a}}$, $\varphi^{\tilde{x}}$ respectively. In particular the scalar fields of the model are $\varphi^{\tilde{x}} = \{\varphi^a, \varphi^s, \varphi^M\}$. The indices $\tilde{a}$, $\tilde{x}$ are flat and curved indices respectively of the $\tilde{n}$-dimensional manifold $\mathcal{M}$ which is parametrized by the scalar fields. This manifold is embedded in an $(\tilde{n}+1)$-dimensional space and is determined by the cubic constraint

$$C_{\tilde{I}\tilde{J}\tilde{K}} h^{\tilde{I}} h^{\tilde{J}} h^{\tilde{K}} = 1 \tag{2}$$

where $\tilde{I}, \tilde{J}, \tilde{K} = 0, \tilde{x}$. $h^{\tilde{I}}$ are functions of the scalar fields defining the embedding of the manifold $\mathcal{M}$. $C_{\tilde{I}\tilde{J}\tilde{K}}$ are constants symmetric in the three indices.

For the model at hand we choose $h^{\tilde{x}} = \varphi^{\tilde{x}}$, where $\varphi^{\tilde{x}}$ are arbitrary and h^0 is to be determined by the constraint given by eq. (2). This choice is convenient since in this case the constraint becomes manifestly $SU(5)$ invariant. Taking the constants $C_{\tilde{I}\tilde{J}\tilde{K}}$ as explained in [29] the constraint reads

$$(h^0)^3 - 3h^0 \left[Tr\mathbf{\Phi}^2 + \frac{1}{2}\chi^\dagger\chi + \frac{1}{2}(\varphi^s)^2 \right] + 4Tr\mathbf{\Phi}^3 - 3\sqrt{\frac{3}{2}}\chi^\dagger\mathbf{\Phi}\chi = 1 \tag{3}$$

where $\mathbf{\Phi} = \varphi^a \mathbf{K}^a$, $a = (1, \ldots, 24)$, is in the adjoint representation of $SU(5)$, in the form of a 5×5 matrix. $\mathbf{K}^a$ denote the $SU(5)$ generators normalized as $Tr(\mathbf{K}^a\mathbf{K}^b) = \frac{1}{2}\delta^{ab}$. χ are scalars in the fundamental representation of $SU(5)$.

The fields φ^M introduced before are just the real and imaginary parts of the fields χ. In particular

$$\varphi^M = (\varphi^A, \varphi^{A+5}), \qquad A = 1, \dots, 5 \tag{4}$$

with

$$\chi^A = \varphi^A + i\varphi^{A+5}, \qquad (\chi^\dagger)^A = \chi^{\bar{A}} = \varphi^A - i\varphi^{A+5}. \tag{5}$$

The bosonic part of the Lagrangian which is $SU(5) \times U(1)_R$ symmetric is given by [29]

$$\begin{aligned} e^{-1}\mathcal{L} &= -\frac{1}{2}R - \frac{1}{4}\mathring{a}_{\tilde{I}\tilde{J}}\mathcal{H}^{\tilde{I}}_{\mu\nu}\mathcal{H}^{\tilde{I}\,\mu\nu} - \frac{1}{2}g_{\tilde{x}\tilde{y}}(\mathcal{D}_\mu\varphi^{\tilde{x}})(\mathcal{D}^\mu\varphi^{\tilde{y}}) \\ &+ \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}\Big\{F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda + \frac{3}{2}gF^I_{\mu\nu}A^J_\rho(f^K_{LF}A^L_\sigma A^F_\lambda) \\ &+ \frac{3}{5}g^2(f^J_{GH}A^G_\nu A^H_\rho)(f^K_{LF}A^L_\sigma A^F_\lambda)A^I_\mu\Big\} \\ &+ \frac{e^{-1}}{4g}\epsilon^{\mu\nu\rho\sigma\lambda}\Omega_{MN}B^M_{\mu\nu}\mathcal{D}_\rho B^N_{\sigma\lambda} \\ &- g^2P - g_R^2P^{(R)}. \end{aligned} \tag{6}$$

The corresponding gauge fields are A^a_μ and $A_\mu = V_0A^0_\mu + V_sA^s_\mu$, where V_0, V_S are constants. The option in which only the $SU(5)$ is gauged is implemented by putting $g_R = 0$. In this case the "s" multiplet is redudant. All scalars are neutral under $U(1)_R$ while under $SU(5)$ we have an adjoint, a $(5 + \bar{5})$ and a singlet. The covariant derivatives appearing in the Lagrangian are both general coordinate and gauge covariant. The fields $\mathcal{H}^{\tilde{I}}_{\mu\nu}$ describe collectivelly the field strengths of the vector fields and the self-dual tensor fields

$$\mathcal{H}^{\tilde{I}}_{\mu\nu} = (F^I_{\mu\nu}, B^M_{\mu\nu}). \tag{7}$$

The constants f_{IJ}^{K} denote the structure constants of the gauge group. f_{ab}^{c} are the usual $SU(5)$ structure constants and $f_{IJ}^{K}=0$ if one of the indices is 0 or s, satisfying thus the condition $V_I f^I_{JK}=0$. g and g_R are the $SU(5)$ and the $U(1)_R$ gauge coupling constants respectively.

The tensor Ω_{MN} in the Lagrangian (6), is the given by the matrix

$$\Omega=\begin{bmatrix} 0 & I_{5\times 5} \\ -I_{5\times 5} & 0 \end{bmatrix}. \tag{8}$$

As far as the tensors appearing in the kinetic terms are concerned, $\mathring{a}_{\tilde{I}\tilde{J}}$ is the restriction of the metric of the $(\tilde{n}+1)$- dimensional space on the $\tilde{n}$-dimensional manifold of the scalar fields and is given by:

$$\mathring{a}_{\tilde{I}\tilde{J}}=-2C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{K}}+3h_{\tilde{I}}h_{\tilde{J}} \tag{9}$$

where

$$h_{\tilde{I}}=C_{\tilde{I}\tilde{J}\tilde{K}}h^{\tilde{J}}h^{\tilde{K}}=\mathring{a}_{\tilde{I}\tilde{J}}h^{\tilde{J}}. \tag{10}$$

In eq. (6) $g_{\tilde{x}\tilde{y}}$ is the metric of the $\tilde{n}$-dimensional manifold $\mathcal{M}$ given by

$$g_{\tilde{x}\tilde{y}}=h^{\tilde{I}}_{\tilde{x}}h^{\tilde{J}}_{\tilde{y}}\mathring{a}_{\tilde{I}\tilde{J}} \tag{11}$$

where $h^{\tilde{I}}_{\tilde{x}}=-\sqrt{\frac{3}{2}}h^{\tilde{I}}{}_{,\tilde{x}}$. Note also that the following relations hold

$$h^{\tilde{I}}h_{\tilde{I}}=1, \quad h^{\tilde{I}}_{\tilde{x}}h_{\tilde{I}}=h^{\tilde{I}}h_{\tilde{I}\tilde{x}}=0 \tag{12}$$

with $h_{\tilde{I}\tilde{x}}=\sqrt{\frac{3}{2}}h_{\tilde{I},\tilde{x}}$.

Supersymmetry invariance requires the existence of additional potential terms in the Lagrangian. In particular the $U(1)_R$ gauging gives rise to the potential $g_R^2 P^{(R)}$ where

$$P^{(R)} = -(P_0)^2 + P_{\tilde{a}} P^{\tilde{a}} \tag{13}$$

with

$$P_0 = 2h^I V_I, \qquad P^{\tilde{a}} = \sqrt{2} h^{\tilde{a}I} V_I. \tag{14}$$

Furthermore the existence of tensor multiplets leads to the appearence of $g^2 P$ where

$$P = 2W^{\tilde{a}} W_{\tilde{a}} \tag{15}$$

with $W^{\tilde{a}}$ given by

$$W^{\tilde{a}} = -\frac{\sqrt{6}}{8} h_M^{\tilde{a}} \Omega^{MN} h_N. \tag{16}$$

Ω^{MN} is the inverse of Ω_{MN} given in (8). The conversion of flat indices ($\tilde{a}$) to curved ones ($\tilde{x}$), and vice-versa, is implemented by the use of $f_{\tilde{x}}^{\tilde{a}}$, the vielbein of the manifold $\mathcal{M}$.

For the specific model the quantities $h_{\tilde{I}}$ are given by

$$\begin{aligned}
h_0 &= (h^0)^2 - \left[Tr\mathbf{\Phi}^2 + \frac{1}{2}\chi^\dagger \chi + \frac{1}{2}(\varphi^s)^2 \right] \\
h_s &= -h^0 \varphi^s \\
h_a &= -h^0 \varphi^a + 4Tr\mathbf{K}^a \mathbf{\Phi}^2 - \sqrt{\frac{3}{2}} \chi^\dagger \mathbf{K}^a \chi \\
h_A &= -\frac{1}{2} h^0 \chi^{\bar{A}} - \sqrt{\frac{3}{2}} (\chi^\dagger \mathbf{\Phi})_A \\
h_{\bar{A}} &= -\frac{1}{2} h^0 \chi^A - \sqrt{\frac{3}{2}} (\mathbf{\Phi}\chi)_{\bar{A}}.
\end{aligned} \tag{17}$$

The metric can be written as (see eq. (9))

$$\mathring{a}_{\tilde{I}\tilde{J}} = b_{\tilde{I}\tilde{J}} + 3h_{\tilde{I}}h_{\tilde{J}} \tag{18}$$

with

$$\begin{array}{rclrcl}
b_{00} & = & -2h^0 & b_{ss} & = & h^0 \\
b_{0a} & = & \varphi^a & b_{ab} & = & h^0\delta^{ab} - 4Tr\left[\{\mathbf{K}^a, \mathbf{K}^b\}\mathbf{\Phi}\right] \\
b_{0A} & = & \frac{1}{2}\chi^{\bar{A}} & b_{aA} & = & \sqrt{\frac{3}{2}}(\chi^\dagger\mathbf{K}^a)_A \\
b_{0\bar{A}} & = & \frac{1}{2}\chi^{A} & b_{a\bar{A}} & = & \sqrt{\frac{3}{2}}(\mathbf{K}^a\chi)_{\bar{A}} \\
b_{0s} & = & \varphi^s & b_{A\bar{A}} & = & \frac{1}{2}h^0\delta^{A\bar{A}} + \sqrt{\frac{3}{2}}\mathbf{\Phi}_{\bar{A}A}.
\end{array} \tag{19}$$

In eq. (19) only the nonvanishing components of $b_{\tilde{I}\tilde{J}}$ are shown. Note that at the point $h^0 = 1, \quad \varphi^{\tilde{x}} = 0$ we have $\mathring{a}_{\tilde{I}\tilde{J}} = \delta_{\tilde{I}\tilde{J}}$ as required by the consistency of the constraint (3).

3 The potential

The part of the potential due to the scalar fields which are non-singlets under the gauge group and do not belong to the gauge multilpets is, see eq. (15),

$$\begin{aligned}
P &= \frac{3}{16}(h^{\tilde{a}}_M\Omega^{MN}h_N)(h^{\tilde{a}}_P\Omega^{P\Sigma}h_\Sigma) \\
&= \frac{3}{16}f^{\tilde{a}\tilde{x}}f^{\tilde{a}\tilde{y}}(h_{M\tilde{x}}\Omega^{MN}h_N)(h_{P\tilde{y}}\Omega^{P\Sigma}h_\Sigma) \\
&= \frac{3}{16}g^{\tilde{x}\tilde{y}}(h_{M\tilde{x}}\Omega^{MN}h_N)(h_{P\tilde{y}}\Omega^{P\Sigma}h_\Sigma).
\end{aligned} \tag{20}$$

Using the complex notation for the fields in the $(5+\bar{5})$ representation we find that

$$h_{M\tilde{x}}\Omega^{MN}h_N = \sqrt{\frac{3}{2}}h_{M,\tilde{x}}\,\Omega^{MN}h_N = -\sqrt{6}i(\chi_{A,\tilde{x}}\,\chi_{\bar{A}} - \chi_{\bar{A},\tilde{x}}\,\chi_A) \tag{21}$$

and thus the non-abelian part of the potential receives the simple form

$$P = -\frac{9}{8}g^{\tilde{x}\tilde{y}}M_{\tilde{x}}M_{\tilde{y}} \tag{22}$$

where the vector $M_{\tilde{x}}$ is given by

$$M_{\tilde{x}} = (\chi_{A,\tilde{x}}\,\chi_{\bar{A}} - \chi_{\bar{A},\tilde{x}}\,\chi_A). \tag{23}$$

From eq. (22) we see that in order to study the potential we need the inverse of the metric $g_{\tilde{x}\tilde{y}}$. With the adopted form of the constraint the metric takes the form

$$g_{\tilde{x}\tilde{y}} = -\frac{3h^0}{(h_0)^2}\left[h_{\tilde{x}}h_{\tilde{y}} + \frac{1}{2}\frac{h_0}{h^0}F_{\tilde{x}\tilde{y}}\right] \tag{24}$$

where

$$F_{\tilde{x}\tilde{y}} = h_{\tilde{x}}b_{0\tilde{y}} + h_{\tilde{y}}b_{0\tilde{x}} - h_0 b_{\tilde{x}\tilde{y}}. \tag{25}$$

The inverse of the metric is then found to be

$$g^{\tilde{x}\tilde{y}} = -\frac{2}{3}h_0\left[F^{\tilde{x}\tilde{y}} - \frac{2h^0(F^{\tilde{x}\tilde{z}}h_{\tilde{z}})(F^{\tilde{y}\tilde{w}}h_{\tilde{w}})}{h_0 + 2h^0(F^{\tilde{z}'\tilde{w}'}h_{\tilde{z}'}h_{\tilde{w}'})}\right] \tag{26}$$

where $F^{\tilde{x}\tilde{y}}$ is the inverse of $F_{\tilde{x}\tilde{y}}$ in (25). In view of the above relations the non-abelian part of the potential can be written as

$$\begin{aligned} P &= \frac{3}{4}\frac{h_0}{h_0 + 2h^0(hF^{-1}h)}\Big\{h_0(MF^{-1}M) \\ &+ 2h^0\left[(MF^{-1}M)(hF^{-1}h) - (MF^{-1}h)^2\right]\Big\}. \end{aligned} \tag{27}$$

The potential stemming from the $U(1)_R$ gauging consists of two terms. The first is negative definite

$$-(P_0)^2 = -4\left[h^0 V_0 + h^s V_s\right]^2 \tag{28}$$

and the second is positive

$$\begin{aligned} P^{\tilde{a}} P_{\tilde{a}} &= 2(h^I_{\tilde{a}} V_I)(h^J_{\tilde{a}} V_J) \\ &= 2g^{\tilde{x}\tilde{y}}(h^I_{\tilde{x}} V_I)(h^J_{\tilde{y}} V_J). \end{aligned} \tag{29}$$

With our choice of coordinates $h^{\tilde{y}}_{\tilde{x}} = -\sqrt{\frac{3}{2}}\delta^{\tilde{y}}_{\tilde{x}}$, and using the relation (12), we find that $h^0_{\tilde{x}} = \sqrt{\frac{3}{2}}\frac{h_{\tilde{x}}}{h_0}$. So

$$P^{\tilde{a}} P_{\tilde{a}} = 3g^{\tilde{x}\tilde{y}} N_{\tilde{x}} N_{\tilde{y}} \tag{30}$$

where the corresponding vectors are given by

$$N_{\tilde{x}} = \left(\frac{h_{\tilde{x}}}{h_0} V_0 - \delta^s_{\tilde{x}} V_s\right). \tag{31}$$

Using the form of $g^{\tilde{x}\tilde{y}}$ found earlier the positive term of the abelian part of the potential is given by

$$\begin{aligned} P^{\tilde{a}} P_{\tilde{a}} &= -\frac{2}{h_0 + 2h^0(hF^{-1}h)}\Big\{V_0^2(hF^{-1}h) - 2V_s V_0 h_0(F^{s\tilde{x}} h_{\tilde{x}}) \\ &+ V_s^2 h_0\left[h_0 F^{ss} + h^0\left(F^{ss}(hF^{-1}h) - (F^{s\tilde{x}} h_{\tilde{x}})^2\right)\right]\Big\}. \end{aligned} \tag{32}$$

The full scalar potential is given by

$$V = g^2 P - g_R^2\left(P_0^2 - P^{\tilde{a}} P_{\tilde{a}}\right) \tag{33}$$

and it is rather involved to deal with analytically. Therefore we will proceed numerically in order to study its vacuum structure.

4 Comments on the vacua

The basic features of our analysis of the scalar potential may be summarized in the following.

(i). If the $U(1)_R$ symmetry is not gauged, then we have a positive definite potential. This potential has flat directions along $< \chi^A >= 0$ which preserve supesymmetry. Thus we have a class of supersymmetric Minkowski vacua in which the gauge group may be broken only by the scalar fields in the adjoint representation. The degeneracy of the vacua may be lifted by radiative corrections. If this is the case the gauge symmetry breaking occurs in the bulk. In the opposite case we have a supersymmetric $SU(5)$ invariant vacuum. [19].

(ii). The $U(1)_R$ gauging induces, as already mentioned, a negative contribution to the potential for supersymmetry to be preserved. This contribution alters the situation drastically as we shall see in the sequel.

In what follows we will study the symmetry breaking patterns $SU(3) \times SU(2) \times U(1)$, $SU(4) \times U(1)$ and $SU(2) \times SU(2) \times U(1)$ occuring since the fields in the adjoint representation may develop non-vanishing v.e.v.'s along the appropriate directions. For the fields χ^A, $\chi^{\bar{A}}$ we take only $\chi^5 = \chi^{\bar{5}}$ to be non-zero, to avoid charge violation.

As a first step we consider the model without the multiplet $(A^s_\mu, \lambda^{si}, \varphi^s)$. This multiplet, unlike the rest, is not necessary in order to construct a locally supersymmetric Lagrangian with $SU(5) \times U(1)_R$ gauge symmetry in five dimensions. In this case we find that the potential has a local maximum at the $SU(5)$ symmetric point, but it develops instability in the directions $\chi^A = \chi^{\bar{A}} = 0$. Along these directions the positive contribution of P disappears and we are left only with $P^{(R)}$. In $P^{(R)}$ the negative term $-(h^0)^2$, with h^0 the real root of the constraint which is continuously connected to unit, dominates the positive term. The instability is caused by the fact that this root keeps growing as $< \varphi^a >$ grows up. Although this sort of instability is not peculiar for these models, see for example [32], it is an unpleasent feature for a realistic model.

In an attempt to lift the instability in a minimal way we proceed to the inclusion of the multiplet $\{A^s_\mu, \lambda^{is}, \varphi^s\}$ which is scalar under $SU(5)$ as we have said. Let us remark at this point that the lowest dimensionality of the scalar manifold arising from string theory is 35 (see [4] Vol. II, page 309). This is the case in the model under consideration after the introduction of the singlet field φ^s. The introduction of an extra $SU(5)$ inert multiplet allows for a combination

$$V_0 A^0_\mu + V_s A^s_\mu \tag{34}$$

for the $U(1)_R$ gauging, and correspondingly modifies the negative term of $P^{(R)}$ to

$$-(V_0 h^0 + V_s \varphi^s)^2 \tag{35}$$

with the appropriate change in the positive term.

In this case it turns out that we still have the instability in the directions

$\chi^A = \chi^{\bar{A}} = 0$. In fact for large values of h^0, φ^s the negative term (35) dominates over the corresponding positive one. Then the $SU(5)$-symmetric point remains either a local minimum or turns to a saddle point.

From the study of the potential we see that with this minimal number of multiplets it is impossible to get rid of the instability if the range of φ^s is infinite. A possible way to remedy the situation is by restricting the field φ^s to take values in a finite range. For instance solitonic solutions of finite variation (kinks) for the scalar fields of five-dimensional supergravity have been long known [34]. Recently explicit examples of such solutions have been worked out for the case of supersymmetric domain wall world embedding in the context of five-dimensional gauged supergravity [30, 35]. Furthermore when 5-d supergravity is obtained from compactification of 11-d supergravity on a smooth Calabi-Yau manifold such neutral scalars emerge naturally [30, 36]. On these grounds it is legitimate and well justified to allow the field φ^s to vary in a finite range.

From the study of the potential we find that for each value of φ^s and V_s there exists a maximum value for V_0 for which stability of the potential is guaranteed. This means that there exist non-trivial combinations of A^0_μ and A^s_μ for the $U(1)_R$ gauging that lead to a stable potential. For instance if we arbitrarily fix the values of φ^s and V_s and choose $V_0 = 0$, the absolute minimum of the potential occurs when all nonsinglet scalars, under $SU(5)$, get vanishing v.e.v.'s. Therefore the vacuum is at the $SU(5)$ symmetric point. The difference now is that we have a negative cosmological constant given by

$$-g_R^2 V_s(\varphi^s)^2.$$

Varying V_0 we find that $SU(5)$ breaking minima develop due to the contribution of h_0 in eq. (35) which is determined by the constaint. The $SU(5)$ symmetric point turns out to be either a local maximum or a local minimum but in the latter case is not the absolute minimum. We have found that after some critical value of V_0 the potential becomes unstable. We point out that at the level of 5-d supergravity the relative values of V_s and V_0 are arbitrary. Therefore it is a matter of the more fundamental theory to determine their values.

In figures 1 and 2 we display the potential for particular values of φ^s, V_s , V_0. Since we are interested in the directions $<\chi^A>=0$, only the abelian part is plotted. In the figure 1 the dashed line corresponds to the direction $SU(2)\times SU(2)\times U(1)$ and the solid line to the $SU(3)\times SU(2)\times U(1)$ direction. The $SU(3)\times SU(2)\times U(1)$ minimum is lower than the $SU(2)\times SU(2)\times U(1)$ one. In the figure 2 we plot the potential in the $SU(4)\times U(1)$ direction. The corresponding minimum is slightly deeper from the phenomenologicaly acceptable $SU(3)\times SU(2)\times U(1)$ vacuum. Note however that the $SU(4)\times U(1)$ direction may be exluded from the geometry of the manifold since in this direction, unlike the other cases, the metric has singularities. Certainly we do not claim that the model selects in this case the right vacuum naturally but the fact that the gauge symmetry is broken as a consequence of the particular $U(1)_R$ gauging is by itself very interesting.

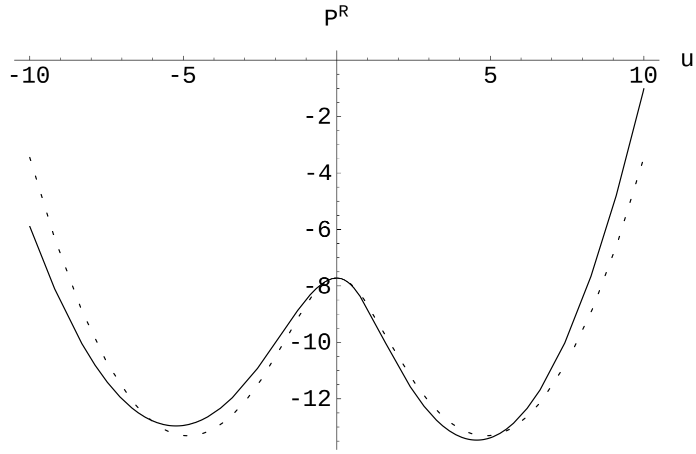

Figure 1: The dashed line shows the potential in the direction $SU(2) \times SU(2) \times U(1)$ while the solid line shows the potential in the direction $SU(3) \times SU(2) \times U(1)$. u denotes the v.e.v of the $\{\mathbf{24}\}$, φ^a, in the corresponding direction. The field φ^s is fixed to unity. The mixing for the $U(1)_R$ gauging is chosen as $V_s = 1$, $V_0 = 0.5$. At the value $V_0 = 0.6$ the potential becomes unstable.

As far as the sypersymmetric nature of the vacua is concerned let us remind that $< \varphi^{\tilde{x}} > \neq 0$ and $< \lambda^{i\tilde{x}} >= 0$ implies for the supersymmetry transformations that $< \delta\lambda^{i\tilde{x}} >= 0$. This in turn implies that in order for the supersymmetry to be preserved we must have $P^{\tilde{a}} = W^{\tilde{a}} = 0$ when the fields are on their vacuum expectation values. Ignoring zero modes of the fünfbein the above relations imply that $M_{\tilde{x}} = N_{\tilde{x}} = 0$. However in the model at hand the vacua which break the gauge symmetry have $P^{\tilde{a}} \neq 0$ and therefore

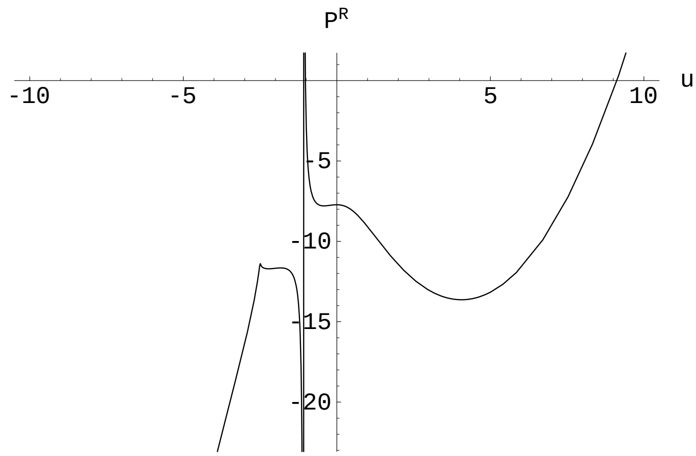

Figure 2: The potential in the direction $SU(4)\times SU(1)$. The singularity of the coordinate system in the negative axis at $u=-1$ is shown. The value of V_0, V_s, φ^s are as in figure 1.

supersymmetry is broken. Let us note that the vacuum expectation values of the scalar fields enter the supersymmetry transformations of the spinors through the combination $gW^{\tilde a}\epsilon^i+\frac{1}{\sqrt2}g_RP^{\tilde a}\delta^{ij}\epsilon_j$ where $\epsilon_1=\epsilon^2,\quad \epsilon_2=-\epsilon^1$. Then supersymmetry for the vacua implies that

$$\begin{aligned} gW^{\tilde a}\epsilon^1 &+ \frac{1}{\sqrt2}g_RP^{\tilde a}\epsilon_2=0\\ gW^{\tilde a}\epsilon^2 &+ \frac{1}{\sqrt2}g_RP^{\tilde a}\epsilon_1=0. \end{aligned}\tag{36}$$

So assuming that $\epsilon^2=\epsilon^1$ the above relations read

$$(gW^{\tilde a}\mp\frac{1}{\sqrt2}g_RP^{\tilde a})\epsilon^1=0.\tag{37}$$

If the vacuum expectation values and the coupling constants are such that either of eq. (37) is satisfied, then we have broken supersymmetry in the bulk space but $N = 1$ supersymmetry in the the four dimensional space. Although very interesting this possibility is ruled out on the grounds that a nonvanishing value of $W^{\tilde{a}} \neq 0$ is required since $P^{\tilde{a}} \neq 0$ too. This adds a positive contribution to the nonabelian part of the potential making these d=4 sypersymmetric vacua not to be the absolute minima of the theory. Thus in general only the $SU(5)$ symmetric point is supersymmetric. In any other case we have simultaneous breaking of both supersymmetry and the gauge symmetry.

The model at hand is by itself an effective one so it may not be inconceivable that φ^s does not vary within a finite range but the dynamics of the underlying theory drives φ^s to a fixed value. In this case the instability is lifted too. A consequence of this is that supersymmetry is completely broken. This is due to the fact that the field φ^s belongs to the same multiplet with the vector field A^s_μ which participates to the $U(1)_R$ gauging. Therefore the mechanism that fixes φ^s to a constant value, in fact sets this to be a moduli field, is intimately connected with the supersymmetry breaking mechanism.

In order to reach to a firm conclusion which one of the aforementioned possibilities for stabilizing the potential can be realized and exploring the supersymmetry properties of the vacua a more detailed analysis of the scalar field manifold is needed. Note that the manifold $\mathcal{M}$ describing this model is neither maximally symmetric nor homogeneous [29] lying therefore outside the cases that have been extensively studied and classified so far [28, 29, 37]. Besides this geometrical analysis an explicit construction of the model from the 11-d supergravity is necessary in order to clarify which one of the particular vacua is favoured.

5 Conclusions

We have studied the classical vacua of a five-dimensional $SU(5)$ supergravity model with the minimal field content and no matter fields. We have found that:

1. Without $U(1)_R$ gauging the potential has flat directions determined by vanishing v.e.v.'s of the fields in the $\mathbf{5}+\bar{\mathbf{5}}$ representation of $SU(5)$. Along these directions supersymmetry is preserved. Certainly this model has to be completed with the matter spectrum. The effective four-dimensional model will arise either after compactification of the fifth dimension or by considering the corresponding action on a three-dimensional brane. In any case the five-dimensional vacuum affects the four-dimensional theory. The fact that at the classical level the gauge symmetry and the supersymmetry breaking does not occur at the five dimensional bulk means that such models are characterized by a large scale of effectiveness (energy desert [38]). The possibility that the degeneracy is lifted by five-dimensional radiative corrections supports models that do not exhibit unification in four dimensions.

2. The $U(1)_R$ gauging creates stability problems. In order to overcome the instability we introduce a vector multiplet neutral under the gauge group. The mechanisms proposed are either the restriction of the variation of the corresponding scalar field, φ^s, in a finite range or fixing this field to a constant value. The former case may be realized by kink solutions existing in the context of five-dimensional supergravity, while the latter is an issue related to the dynamics of the uderlying fundamental theory. Under these assumptions we have found that the potential becomes stable in a certain range of the parameters V_0, V_s, which determine the mixing of the graviphoton and the vector field of the extra multiplet which plays the rôle of the $U(1)_R$ gauge field. The vacua in this case are of the anti-de-Sitter type. As far as the fate of the gauge symmetry is concerned the choice $V_0 = 0$ yields an $SU(5)$ invariant minimum. In this case supersymmetry is unbroken if φ^s varies in a finite range but is broken if φ^s is fixed to a constant value. When $V_0 \neq 0$, $SU(5)$ breaking minima appear. In all the minima where the gauge symmetry is broken supersymmerty is also broken. As is already quoted the five-dimensional vacuum determines the scale of effectiveness of the corresponding brane-world model. For example the $SU(5)$ breaking vacua are compatible with the scenaria with low energy scale brane description. The nonsupersymmetric $SU(5)$ vacuum, may be compatible with unification with low string scale [39].

Thus we see that the scenaria for physics beyond the SM may in principle be incorporated in a unified five-dimensional supergravity content. Certainly the properties of the scalar field manifold, the mechanism for stabilization of the potential in the case of anti-de-Sitter space and the details of including matter on the brane describing the four-dimensional space require further investigation. Work towards this direction is in progress.

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References

[1] H. Georgi and S. Glashow, *Phys. Rev. Lett.* **32** 438 (1974),
H. Georgi, H. Quinn and S. Weinberg, *Phys. Rev. Lett.* **33** 451 (1974),
A.J. Buras, J.R. Ellis, M.K. Gaillard, D.V. Nanopoulos, *Nucl. Phys.* **B135**, 66 (1978),
S. Dimopoulos, S. Raby and F. Wilczek, *Phys. Rev.* **D24**, 1681 (1981),
S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).

[2] L. Ibanez and G.G. Ross, *Phys. Lett.* **B106**, 439 (1981),
N. Sakai, *Z. Phys.* **C11** 153 (1981),
U. Amaldi, W. de. Boer and H. Furstenau, *Phys. Lett.* **B260**, 447 (1991).

[3] M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory* (Cambridge U. Press, 1987).

[4] J. Polchinski, *String Theory* (Cambridge U. Press, 1998).

[5] For a review see K.R. Dienes, *Phys. Rep.* **287**, 447 (1997).

[6] I. Antoniadis, C. Bachas, D. Lewellen and T. Tomaras, *Phys. Lett.* **B207**, 441 (1988),
C. Kounnas and M. Porrati, *Nucl. Phys.* **B310**, 355 (1988),
S. Ferrara, C. Kounnas and M. Porrati, and F. Zwirner, *Nucl. Phys.* **B318**, 75 (1989),
I. Antoniadis,*Phys. Lett.* **B246**, 377 (1990).

[7] E. Witten, *Nucl. Phys.* **B443**, 85 (1995) D. Olive and P. West eds. *Duality and Supersymmetric Theories* (Cambridge U. Press, 1998).

[8] For a review see C.P. Bachas, hep-ph/0003259.

[9] P. Hořava and E. Witten, *Nucl. Phys.* **B460**, 506 (1996); E. Witten, *ibid.* **B471**, 135 (1996).

[10] J. Lykken, *Phys. Rev.* **D54**, 3693 (1996)

[11] G. Shiu and S.-H. H. Tye, *Phys. Rev.* **D58**, 106007 (1998)

[12] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B429**, 263 (1998),
I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, *Phys. Lett.* **B436**, 263 (1998).

[13] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Phys. Rev.* **D59**, 086001 (1999).

[14] Z. Kakuzhadse and S.-H. H. Tye, *Nucl.Phys.* **B548**, 180 (1999).

[15] For a review see I. Antoniadis and K. Benakli, hep-ph/0007226.

[16] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** 3370 (1999)

[17] L. Randall and R. Sundrum, *Phys. Rev. Lett.* **83** 4690 (1999)

[18] For a review see V.A. Rubakov, hep-ph/0104152.

[19] E. A. Mirabelli and M. Peskin, *Phys. Rev.* **D58**, 0605002 (1998)

[20] K. R. Dienes, E. Dudas and T. Gherghetta *Phys. Lett.* **B436**, 55 (1998); *Nucl.Phys.* **B537**, 47 (1999).

[21] For a review see E. Dudas, *Class. Quant. Grav.* **17**, R41 (2000).

[22] I. Antoniadis, E. Kiritsis, T.T. Tomaras, *Phys. Lett.* **B486**, 186 (2000)

[23] Y. Kawamura, *Prog. Theor. Phys.* **105**, 691 (2001) *ibid* 99 (2001).

[24] G. Altarelli and F. Feruglio, *Phys. Lett.* **B511**, 257 (2001).

[25] A. Hebecker and J. March-Russell, hep-ph/0106166.

[26] A. Pomarol, *Phys. Rev. Lett.* **85**, 4004 (2000).

[27] T. Gherghetta and A. Pomarol, *Nucl.Phys.* **B602**, 3 (2001).

[28] B.de Wit, H. Nicolai, *Nucl.Phys.* **B208**, 323 (1982),
M. Pernici, K. Pilch, P. van Niuwenhuizen, *Phys. Lett.* **B143**, 103 (1984);

Phys. Lett. **B154**, 268 (1985); *Nucl.Phys.* **B272**, 598 (1986),

M. Pernici, K. Pilch, P. van Niuwenhuizen, *Nucl.Phys.* **B259**, 460 (1985),

M. Günaydin, G. Sierra, P.K. Townsend, *Nucl.Phys.* **B242**, 244 (1984); *Phys. Lett.* **B133**, 72 (1983); *Phys. Rev. Lett.* **B53**, 332 (1984); *Phys. Lett.* **B144**, 41 (1984).

[29] M. Günaydin, M. Zagermann, *Nucl.Phys.* **B572**, 131 (2000)

[30] A. Lukas, B.A. Ovrut, K.S. Stelle and D. Waldram, *Nucl. Phys.* **B552**, 246 (1999),

J.R. Ellis, Z. Lalak, S. Pokorski and S. Thomas, *Nucl. Phys.* **B563**, 107 (1999),

A. Ceresole, G. Dall'Agata, *Nucl.Phys.* **B585**, 143 (2000),

A. Falfowski, Z. Lalak and S. Pokorski, *Phys. Lett.* **B509**, 337 (2001).

E. Bergshoeff, R. Kallosh, A. Van Proeyen, *JHEP* **0010**, 033 (2000).

[31] For a review see P. Fré, hep-th/0102114.

[32] M. Günaydin, M. Zagermann, *Phys. Rev.* **D62**, 044028 (2000).

[33] J. Ellis, M. Günaydin, M. Zagermann, Options for Gauge Groups in Five-Dimensional Supergravity, [hep-th/0108094].

[34] M. Günaydin, G. Sierra, P.K. Townsend, *Class. Quantum Grav.* **3** 763 (1986).

[35] K. Behrndt, M. Cvetic, *Phys. Lett.* **B475** 253 (2000).

[36] T. Mohaupt, M. Zagermann, Gauged Supergravity and Singular Calabi-Yau Manifolds, [hep-th/0101055].

[37] B. de Wit and A. Van Proeyen, *Commun. Math. Phys.* **149**, 307 (1992).

[38] C. Bachas, hep-th/0001093.

[39] C.P. Bachas, *JHEP* **9811** 023 (1998).